

Transfer of single photon polarization states by two-channel continuous variable teleportation

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Superpositions of two orthogonal single-photon polarization states are commonly used as optical qubits. If such qubits are sent by continuous variable quantum teleportation, the modifications of the qubit states due to imperfect entanglement cause an increase in the average photon number of the output state. This effect can be interpreted as an accidental quantum cloning of the single photon input. We analyze the output statistics of the single photon teleportation and derive the transfer and cloning fidelities from the equations of the polarization qubit.

Keywords: quantum teleportation, continuous variables, polarization, cloning fidelity.

I. INTRODUCTION

Quantum teleportation is a method for transferring quantum states to a remote location using quantum entanglement and classical communication. The original proposal by Bennett et al. [1] was based on a discrete set of basis states, and its implementation for the two level system of single photon polarization was experimentally realized by Bouwmeester et al.[2]. Vaidman [3] pointed out that it is also possible to transfer quantum states using continuous variable entanglement in its singular case. Later, continuous variable (CV) teleportation was adapted to the physically realistic non-singular case by Braunstein et al. [4] and the implementation using entanglement obtained by optical squeezing was experimentally realized by Furusawa et al. [5].

The transfer of two level systems such as the single photon polarization state teleported in ref. [2] is of particular significance for quantum information because it represents a qubit, the smallest unit of quantum information. It may therefore be of interest to consider the transfer of photon polarization qubits using the CV teleportation scheme reported in ref. [5]. As we discussed previously [6], this can be achieved by separately applying the standard CV teleportation protocol to a pair of orthogonally polarized modes. Our discussion in ref.[6] showed that this method can result in different output photon numbers in the teleported state. The teleported photon can either be lost, resulting in a vacuum output component, or it can be multiplied, resulting in an N -photon output state. The latter effect can be interpreted as an accidental quantum cloning procedure applied to the single photon input.

In our previous work, we derived the transfer operator

for CV teleportation [7] and applied it to single photon teleportation problems [6]. In the following, we analyze the output statistics for the qubit teleportation in terms of the transfer and cloning fidelities for different photon number outputs. It is shown that the cloning fidelities approach their optimal values for highly entangled states.

II. TWO-MODE TRANSFER OPERATOR FORMALISM

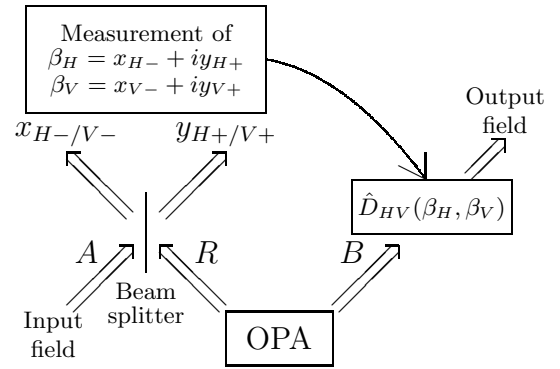


FIG. 1: Schematic representation of the two-mode quantum teleportation setup. The entangled state is generated by four mode squeezing in an optical parametric amplifier (OPA). Four separate homodyne detection measurements are used to obtain the polarization components of the complex displacement amplitudes β_H and β_V .

Fig.1 shows the schematic setup of the CV quantum teleportation for the two polarization modes of the input

field A . Each local light field is now described by two polarization modes. An optical parametric amplifier (OPA) generates the squeezed state entanglement between the R and B modes in both the horizontal (H) and the vertical (V) polarizations, resulting in a pair of beams equally entangled in every possible polarization direction. This four-mode squeezed entangled state can be written as

$$\begin{aligned} |\text{EPR}(q)\rangle_{R,B} = & \\ (1-q^2) \sum_{n_H, n_V=0}^{\infty} q^{(n_H+n_V)} & |n_H; n_V\rangle_R \otimes |n_H; n_V\rangle_B, \end{aligned} \quad (1)$$

where R is the mode used by Alice (sender) as a quantum reference in the joint measurement of A and R , and B is the output mode on Bob's side (receiver). The variable q defines the degree of squeezing, and thus the amount of entanglement of the state. It may be worth noting that this entangled state can be written as a product state of two entangled states for any pair of orthogonal polarizations.

It is now possible to derive a transfer operator $\hat{T}_{\text{pol.}}(\beta_H, \beta_V)$ for the two mode teleportation, so that the output state in B is given by [7]

$$|\psi_{\text{out}}(\beta_H, \beta_V)\rangle_{HV} = \hat{T}_{\text{pol.}}(\beta_H, \beta_V) |\psi_{\text{in}}\rangle_{HV}. \quad (2)$$

As has been shown in ref.[6], this transfer operator can be obtained by a product of two single mode transfer operators. Using the results of ref.[7, 8], the transfer operator $\hat{T}_{\text{pol.}}$ can therefore be expressed using the Fock states $|n_H; n_V\rangle_{HV}$ and the displacement operators $\hat{D}_{HV}(\beta_H, \beta_V)$ can therefore be expressed in its diagonal form by the displaced Fock states $\hat{D}_{HV}(\beta_H, \beta_V) |n_H; n_V\rangle_{HV}$,

$$\begin{aligned} \hat{T}_{\text{pol.}}(\beta_H, \beta_V) &= \hat{T}_{H,q}(\beta_H) \otimes \hat{T}_{V,q}(\beta_V) \\ &= \frac{1-q^2}{\pi} \sum_{n_H=0}^{\infty} \sum_{n_V=0}^{\infty} q^{n_H+n_V} \\ &\times \hat{D}_{HV}(\beta_H, \beta_V) |n_H; n_V\rangle \langle n_H; n_V| \hat{D}_{HV}(-\beta_H, -\beta_V). \end{aligned} \quad (3)$$

In principle we can decompose the polarization transfer operator $\hat{T}_{\text{pol.}}$ into any two orthogonal modes (e.g. right and left circular polarization mode, $\pm 45^\circ$ polarization mode). The choice of horizontal (H) and vertical (V) polarization is made to simplify the labeling.

We can now apply this transfer operator to the specific case of single photon inputs. For an unknown polarization, the input state is described by a superposition of the two basis states, $c_H |1; 0\rangle + c_V |0; 1\rangle$. However, as noted above, the choice of the polarization basis is arbitrary. Therefore, all polarization states are teleported equally well, and it is sufficient to derive the output statistics for the horizontally polarized input $|1; 0\rangle$. As explained in

more detail in ref.[6], the results for the fidelities and the output photon number distributions will be the same for any other input polarization.

The output state for the teleportation of a single horizontally polarized qubit reads [6]

$$\begin{aligned} |\psi_{\text{out}}\rangle_{HV} &= \hat{T}_{H,q}(\beta_H) |1\rangle \otimes \hat{T}_{V,q}(\beta_V) |0\rangle \\ &= \frac{1-q^2}{\pi} e^{-(1-q^2)(|\beta_H|^2 + |\beta_V|^2)/2} \\ &\times \hat{D}_{HV}((1-q)\beta_H, (1-q)\beta_V) \\ &((1-q^2)\beta_H^* |0; 0\rangle + q |1; 0\rangle). \end{aligned} \quad (4)$$

Since this state is essentially a product state of the vacuum teleportation in the vertical polarized component and the one photon teleportation in the horizontally polarized component, we can now determine the photon statistics of the output using the results derived in ref.[6].

III. CONTINUOUS VARIABLE TELEPORTATION OF PHOTON POLARIZATION

The average numbers of horizontally and vertically polarized photons in the output of the teleportation can be determined by averaging the results given in eq. (4) over all measurement values of β_H and β_V [6]. The results read

$$\begin{aligned} \langle n_H \rangle &= \frac{2}{1+q}, \\ \langle n_V \rangle &= \frac{1-q}{1+q}. \end{aligned} \quad (5)$$

The effect of CV teleportation on the average photon numbers is simply given by an addition of $(1-q)/(1+q)$ photons to each channel. This additional intensity originates from the “quantum duty” paid for non-maximal entanglement, as explained by Braunstein et al. in the original proposal of optical CV teleportation [4]. Since the additional photons are unpolarized, the polarization fidelity of the output photons is reduced to

$$F_{\text{av.}} = \frac{\langle n_H \rangle}{\langle n_H \rangle + \langle n_V \rangle} = \frac{2}{3-q}. \quad (6)$$

As expected, this value increases from the classical teleportation limit of $2/3$ at no entanglement ($q = 0$) to a perfect fidelity of one at maximal entanglement ($q = 1$).

The effect of CV teleportation on the average polarization fidelity $F_{\text{av.}}$ is therefore easily explained by the “quantum duty” according to Braunstein et al.[4]. However, this result does not show how the teleportation errors and the output photon number are correlated. We can now apply our more detailed results from ref.[6] to derive this correlation and to identify the origin of the polarization errors. As shown in ref.[6], the output photon number distribution of the single-mode teleportation

of a single photon input is

$$\begin{aligned} p_q^1(n) &= \int d^2\beta |\langle n | \hat{T}_q(\beta) | 1 \rangle|^2 \\ &= \frac{1+q}{2} \left(\frac{1-q}{2} \right)^{n+1} \left(1 + n \left(\frac{1+q}{1-q} \right)^2 \right). \end{aligned} \quad (7)$$

In the case of a zero photon input, the output photon number distribution corresponds to a thermal distribution with

$$\begin{aligned} p_q^0(n) &= \int d^2\beta |\langle n | \hat{T}_q(\beta) | 0 \rangle|^2 \\ &= \frac{1+q}{2} \left(\frac{1-q}{2} \right)^n. \end{aligned} \quad (8)$$

Since the teleportation of the horizontally polarized photon given by eq.(4) can be written as a product of a single photon polarization in H and a vacuum teleportation in V , the output statistics of the two polarizations is given by the product of the probabilities given above. The joint probability thus reads

$$\begin{aligned} p^{1,0}(n_H, N - n_H) \\ = \left(\frac{1+q}{2} \right)^2 \left(\frac{1-q}{2} \right)^{N+1} \left(1 + n_H \left(\frac{1+q}{1-q} \right)^2 \right), \end{aligned} \quad (9)$$

where $N = n_H + n_V$ is the total photon number in the output.

It is now possible to determine the polarization fidelities for a specific number of output photons. For example, it is possible to post-select the cases where only a single photon is found in the output. As previously mentioned in [6], the conditional polarization fidelity F_1 for this post-selected output is given by

$$F_1 = \frac{p^{1,0}(1,0)}{p^{1,0}(1,0) + p^{1,0}(0,1)} = \frac{2(1+q^2)}{2(1+q^2) + (1-q)^2}. \quad (10)$$

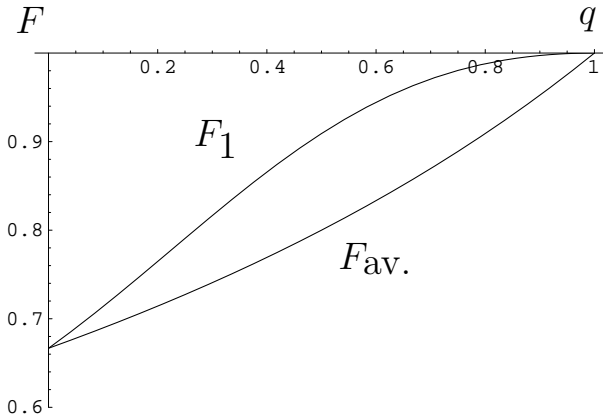


FIG. 2: Comparison of the average polarization fidelity F_{av} and the post-selected fidelity F_1 of the single photon output for variable entanglement parameter q .

Fig.2 shows the comparison between this post-selected fidelity and the average polarization fidelity F_{av} . It can be seen that F_1 is higher than F_{av} for all values of the entanglement parameter q . We can therefore conclude that the errors that decrease the fidelity to F_{av} originate from multi photon outputs ($N > 1$). As we will discuss in the next section, this result can be interpreted in terms of the limits on quantum cloning imposed on the cloning fidelities F_N of the multi photon outputs.

IV. CLONING FIDELITY

Since continuous variable teleportation does not preserve the input photon number, it is possible to detect more than one output photon in the teleportation of a single input photon. Specifically, the probability $P_q(N)$ of obtaining N photons in the output is given by

$$\begin{aligned} P_q(N) &= \sum_{n_H=0}^N p^{1,0}(n_H, N - n_H) \\ &= (N+1) \left(\frac{1+q}{2} \right)^2 \left(\frac{1-q}{2} \right)^{N+1} \left(1 + \frac{N}{2} \left(\frac{1+q}{1-q} \right)^2 \right). \end{aligned} \quad (11)$$

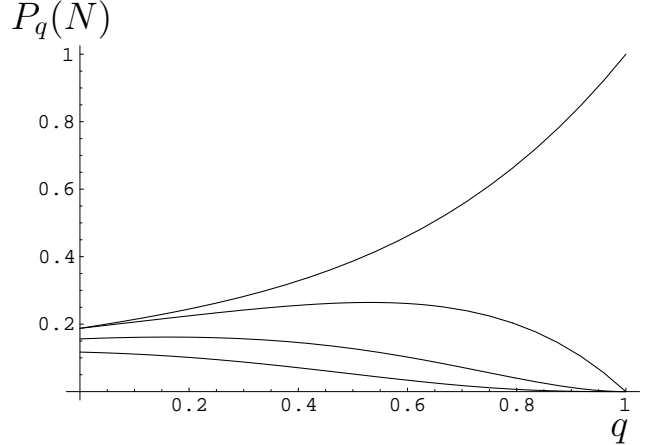


FIG. 3: Probability of obtaining N -photon output as a function entanglement parameter q . From top to bottom, the curves show $P_q(1)$, $P_q(2)$, $P_q(3)$ and $P_q(4)$.

Fig.3 shows the probabilities for $N = 1$, $N = 2$, $N = 3$, and $N = 4$ output photons. For finite entanglement ($q < 1$), there is always a non-vanishing chance of generating additional photons. However, the probabilities of obtaining high photon numbers N decrease rapidly with increasing entanglement q , consistent with the decrease in the average output photon number due to the reduced “quantum duty”.

Since the output photons are indistinguishable, the polarization state of the input photon is transferred equally to all of them. It is therefore possible to consider all of

the N output photons as clones of the original input photon. The conditional probability of finding any one of the N photons in the same polarization as the input state is therefore equal to the cloning fidelity F_N . This fidelity can be determined from the joint probabilities given in eq. (9). It reads

$$F_N = \left(\frac{2}{3} + \frac{\left(\frac{1+q}{1-q} \right)^2 - 1}{3N \left(\frac{1+q}{1-q} \right)^2 + 6} \right). \quad (12)$$

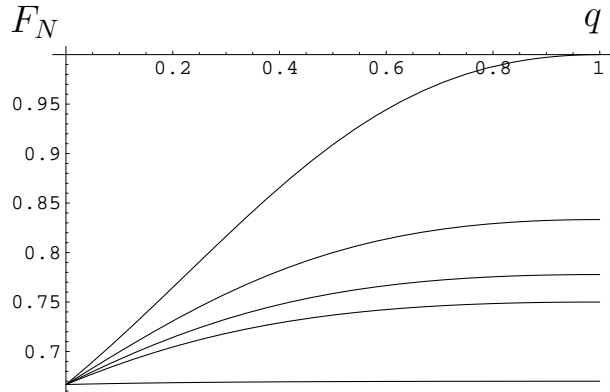


FIG. 4: Cloning fidelities F_N for different output photon numbers N . From top to bottom, the curves show F_1, F_2, F_3, F_4 and F_{100} .

Fig.4 shows the single photon fidelity F_1 and the cloning fidelities for two to four output photons. In addition, the high N limit is indicated by the result for $N = 100$, which is basically indistinguishable from a flat line at $F_\infty = 2/3$. As eq.(12) shows, all cloning fidelities F_N are equal to $2/3$ for classical teleportation at $q = 0$. The fidelities increase with q and reach their maximal value at $q = 1$. However, the limit for $q \rightarrow 1$ cannot be one for $N > 1$, because it would contradict the no-cloning theorem [9]. Therefore, the upper limit depends on N according to

$$\frac{2}{3} \leq F_N \leq \frac{2N+1}{3N}. \quad (13)$$

This upper limit is equal to the maximal cloning fidelity possible for a cloning protocol producing N output photons from one input photon. For an infinite number of clones, this limit is equal to the teleportation fidelity of classical teleportation, as indicated by the flat line for F_{100} in fig. 4.

V. CONCLUSIONS AND OUTLOOK

We have analyzed the CV teleportation of a single photon polarization state by applying the transfer operator formalism to two orthogonally polarized teleportation channels. The modifications of the output states due to imperfect entanglement result in an increase in the average photon number in the output due to the “quantum duty” paid for the use of non-maximal entanglement. The average polarization fidelity of all output photons is therefore reduced by the random polarization of this “quantum duty”. However, our results also show that the conditional polarization fidelity of the single photon transfer is much higher than the average fidelity, indicating that the polarization errors are greater in the multi-photon outputs. Since the transfer of a single photon to a multi-photon output corresponds to a quantum cloning process, this observation can be understood as a result of the no cloning theorem. Indeed, our detailed analysis shows that the cloning fidelities for each N photon output approaches the maximal possible value for quantum cloning protocols as the entanglement parameter q approaches one. A significant part of the polarization errors in the output photons thus originates from the cloning errors associated with the accidental generation of additional output photons.

From the quantum communication point of view, it seems to be quite remarkable that CV teleportation can produce nearly optimally cloned qubits as an accidental side effect of qubit teleportation. In this context, the relation with intentional telecloning protocols [10] may be of further interest.

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